Chaos

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ABSTRACT

Finding an optimal strategy at a minimum cost to efficiently disintegrate a harmful network into isolated components is an important and interesting problem, with applications in particular to anti-terrorism measures and epidemic control. This paper focuses on optimal disintegration strategies for spatial networks, aiming to find an appropriate set of nodes or links whose removal would result in maximal network fragmentation. We refer to the sum of the degree of nodes and the number of links in a specific region as region centrality. This metric provides a comprehensive account of both topological properties and geographic structure. Numerical experiments on both synthetic and real-world networks demonstrate that the strategy is significantly superior to conventional methods in terms of both effectiveness and efficiency. Moreover, our strategy tends to cover those nodes close to the average degree of the network rather than concentrating on nodes with higher centrality.

Beneficial networks are the most frequently encountered class of networks. We hope to ensure their continuous, stable, and effective operation through various means, such as optimized design and coordinated control. However, we also need to find efficient strategies to break harmful networks down. With these aims in mind, this paper introduces a method based on region centrality for disintegration strategies in spatial networks.

I. INTRODUCTION

Over the last few decades, there has been increasing interest in the behavior of complex networks, particularly with regard to exploring the general laws of the networks that are widespread in nature and society.[1,2] Although networks can be considered to be like transportation systems that make the world more closely connected, with all the advantages that this provides, there are also undesirable aspects of this connectivity. The most important examples of this are terrorist and disease transmission networks.[3,4] How to effectively disintegrate such harmful networks through an optimal removal strategy has become an urgent problem.[5,6]

The impact on network integrity of the removal of a single node is limited. However, if multiple nodes or links in a network are removed, this may divide the network into several independent internally connected components.[7,8] Therefore, the effectiveness of network disintegration is influenced by network structure and depends on the removal strategy employed. Albert *et al.*[9] and Holme *et al.*[10] found that scale-free networks are robust to "random removal" but are vulnerable to "intentional attacks" based on degree centrality. As the essence of network disintegration is a combinatorial optimization problem, there have been proposals, mainly from the field of operations research, that the optimal network disintegration solution could be found by solving a mathematical programming model.[11,12] In the case of large-scale networks, attempts can be made to remove nodes (links) one by one in descending order by calculating the centrality of nodes in terms, for example, of degree,[9,10,13] betweenness,[10,14] k-core,[15] closeness,[16] and subgraph centrality.[17] However, a set composed of a single important node (link) may not be the essential set of nodes (links). Therefore, alternative network disintegration methods have been proposed that are based on heuristic algorithms. Morone and Makse[18] and Mugisha and Zhou[19] introduced the collective influence (CI) algorithm, which uses the direct and indirect neighbors of each node in a specific range to obtain a quantitative measurement of the influence of the nodes and then eliminates nodes based on the descending order of their CI values. Zhou[20] and Mugisha and Zhou[19] considered a belief propagation-guided decimation (BPD) probability model to measure the removal probability of each node in a network. Other heuristic algorithms have also been introduced to search for key players in a network, such as a branch removal algorithm,[21] CoreHD,[19,22] and a min-sum algorithm.[23] In recent years, many evolutionary algorithms have been applied to the network disintegration problem. Wu's group[24–27] studied the optimal disintegration strategies of complex networks for undirected, directed, and multilayer networks based on tabu search. Ventresca[28] presented a simulated annealing algorithm to solve the network disintegration problem. Deng *et al.*[29] developed a disintegration optimization approach based on a genetic algorithm with a cost constraint model. With the rapid development of machine learning, Fan *et al.*[30] introduced a deep reinforcement learning algorithm into the network disintegration problem and proposed the FINDER algorithm.

The spatial network is a topological network embedded in the Euclidean plane. It is composed of nodes and links with geospatial attributes, and its topological characteristics are no different from the original topological network.[31,32] There has also been extensive study of the invulnerability of spatial networks. In particular, research on infrastructure networks based on real geographic attributes has developed rapidly. Neumayer and co-workers[33,34] proposed several methods to deal with purely geometric problems, such as those concerning link position, length, and mutual relationships, in the context of the American fiber network and provided an effective method to identify critical areas of this complex spatial network. Agarwal *et al.*[35] presented a new hippodrome model based on Neumayer's critical region recognition algorithm and further proposed a new general probabilistic model based on this method.[36] Peng *et al.*[37] considered a method to search key nodes and critical areas based on network node failure. Deng *et al.*[27] introduced the use of the tabu search algorithm into the spatial network disintegration problem.

The studies listed above focus on finding the vital links or nodes rather than building a model to evaluate the invulnerability of the spatial network. Furthermore, the majority of previous studies of network disintegration strategies with spatial information have considered the destruction mode on small-scale networks. However, the scale of network systems has increased dramatically in recent decades, and conventional algorithms may have too high a computational cost or may suffer from deterioration in performance when applied to such large-scale networks. Inspired by the centrality measures applied in topology networks, here we extend them to spatial networks and propose regional centrality. We define region centrality as the sum of centrality measures of all nodes or links in a specific area. In this paper, we introduce a disintegration strategy based on region centrality to search for the critical areas of a network at a minimal increase in time cost compared with that of a conventional spatial network optimization model with multiple disintegration circles. This strategy allows linear processing for node sets or link sets in the disintegration circles and is also applicable to large-scale networks. Furthermore, to more clearly illustrate the underlying principles of this disintegration strategy and the network changes to which it leads, we introduce several additional metrics to explore the node features of the strategy.

The remainder of the paper is organized as follows. In Sec. II, we provide the definitions of the spatial network disintegration model with multiple disintegration circles. In Sec. III, the optimal disintegration strategy in a spatial network based on region centrality is explained. In Sec. IV, this strategy is applied to several synthetic network models. We then evaluate the strategy on two real-world networks in Sec. V. Finally, in Sec. VI, we conclude the work and discuss some possible extensions.

II. SPATIAL NETWORK DISINTEGRATION MODEL WITH MULTIPLE DISINTEGRATION CIRCLES

A. Spatial network disintegration model

In simple terms, a network is a collection of nodes joined together in pairs by links. It can be described as an undirected and unweighted graph $G = (V, E)$, where V is a finite nonempty set of nodes and $E \subseteq \{(u, v) \mid u, v \in V\}$ is the set of links between the nodes. Let $N = |V|$ be the number of nodes, where the different nodes are labeled as $v_1, v_2, \ldots, v_N$. The degree of the ith node in the network is denoted by k_i. We denote the number of links by $W = |E|$, which represents the total number of links. The most common way to represent the basic structure of a graph is the adjacency matrix $A(G) = (a_{ij})_{N\times N}$, its elements being $a_{ij} = a_{ji} = 1$ if node v_i and node v_j are connected.

We map the network in a coordinate system and define $(c_{ij})_{N\times 2}$ as the coordinate matrix of G, where (c_{i1}, c_{i2}) are the coordinates of node v_i.[27] To facilitate data processing, we use normalized network coordinates to build spatial network disintegration models as follows:

$$\widetilde{c}_{ij} = \frac{c_{ij} - \min(C(:, j))}{\max(C(:, j)) - \min(C(:, j))}, \quad j = 1, 2. \tag{1}$$

Next, we assume the disintegration area of the spatial network to be a circle o and then define the coordinate matrix of this circle as follows:

$$O(G) = (o_{k1}, o_{k2}), \quad k = 1, 2, 3, \ldots, K, \tag{2}$$

in which (o_{k1}, o_{k2}) are the center coordinates of the circle o_k, and K is the number of circles. We also assume that the radius of the disintegration circle is r. We remove from the network all nodes that lie in the circle and those links that are connected to these nodes.

The location of several disintegration circles in the network should be determined first, based on normalized network coordinates. It is a very time-demanding computation to search for multiple optimal center positions. In this paper, we assume that the spatial network coordinate system is divided into $w \times w$ grids, where the center of the disintegration circle is located at the intersection of the grids. Therefore, there are $M = w^2$ optional positions for the center of the circle, where M represents the number of potential positions at which the center can be placed.

Based on the above assumptions, we define the spatial network disintegration strategy with multiple disintegration circles as $X = [x_1, x_2, \ldots, x_M]$, its elements being $x_i = 1$ if the corresponding ith circle belongs to $O(G)$, and $x_i = 0$ otherwise. We then get the number of disintegration circles as $K = \sum_{i=1}^{M} x_i$. Moreover, we assume that $\hat{V} \subseteq V$ is the set of nodes located in multiple disintegration circles, $n = |\hat{V}|$ is the number of removed nodes, and

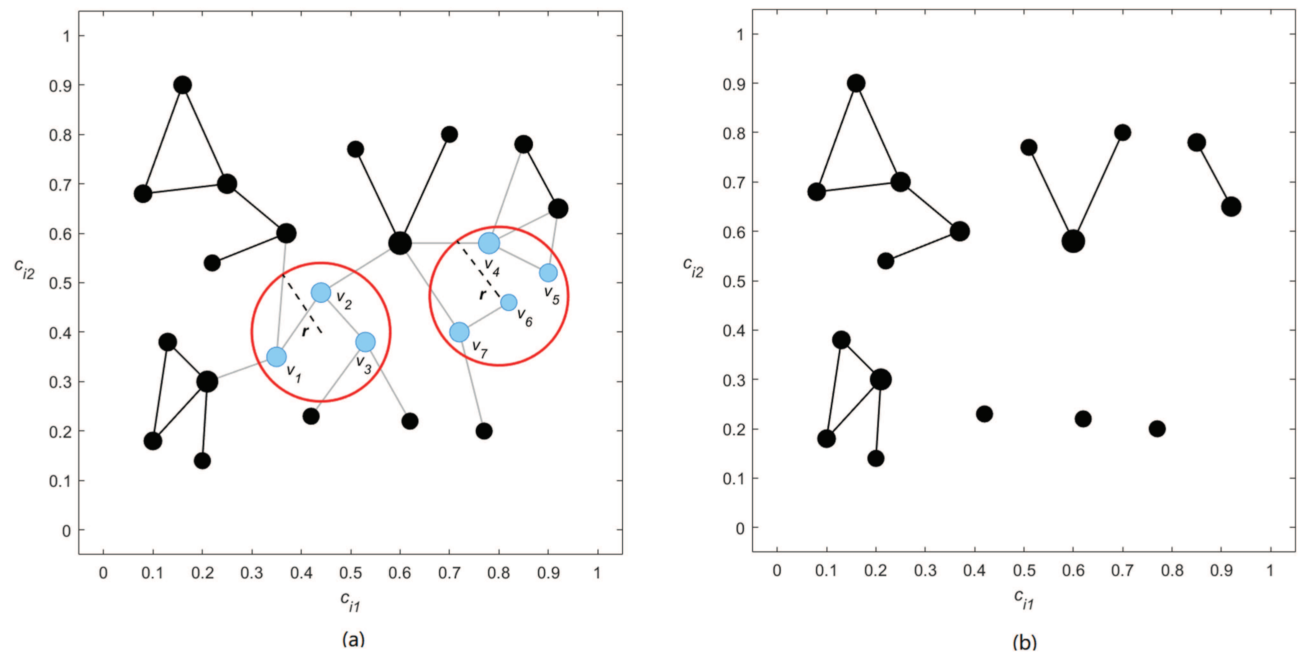

FIG. 1. Schematic of the spatial network disintegration model with multiple disintegration circles: (a) initial network, where the red circles represent the disintegration circles with radius r and the blue nodes represent the nodes to be removed; (b) network remaining after disintegration.

$\hat{G} = (V - \hat{V}, \hat{E})$ is the network that remains after removal of nodes in $\hat{V}$.

Figure 1 displays the process of disintegrating the spatial network using this method. The network contains 24 nodes and 27 links and has been embedded in the normalized network coordinate system. We randomly selected two disintegration circles with center coordinates $(0.44, 0.40)$ and $(0.800, 0.473)$, respectively, and radius r. The nodes v_1, v_2, v_3, v_4, v_5, v_6, and v_7 are located in the disintegration circles, and so these nodes and the links connected to them will be removed from the network.

B. The measure function

1. Metrics to measure the effect of disintegration

In this paper, we denote by Γ the function that measures network performance. The removal of essential nodes may have a significant impact on the network, and so network performance decreases as a result of the network disintegration process. We assume that the disintegration effect is determined by the difference between the measure function of the initial network and that of the network after node removal, i.e., $\Phi(X) = \Gamma(G) - \Gamma(\hat{G}) \geq 0$. In summary, the size of $\Phi(X)$ reflects the destructive effect of the network disintegration strategy.

There are many ways to quantify the structure of a network. One key quantity is the relative size of the largest (giant) connected component (LCC), which is defined as follows:

$$LCC = \frac{C(Q)}{N}, \tag{3}$$

where $C(Q)$ is the number of nodes in the LCC of the network after removal of the node set Q. As the number of removed nodes increases, the network will eventually degenerate into many disconnected subgraphs, and so the relative size of the LCC will gradually decrease until the LCC vanishes, i.e., $LCC = 0$.

Second, we chose natural connectivity (NC) to measure network performance. This metric is a way to quantify the spectral robustness of a complex network, and it has an exact physical meaning and a simple mathematical formula.[38] The average eigenvalue of the network adjacency matrix is defined as follows:

$$NC = \ln\left(\frac{1}{N}\sum_{i=1}^{N} e^{\lambda_i}\right), \tag{4}$$

where λ_i is the ith largest eigenvalue of the adjacency matrix $A(G)$.

We aim to achieve the best effect of network destruction by searching for the optimal position of K disintegrating circles. Therefore, the spatial network disintegration model with multiple disintegration circles can be expressed as follows:

$$\max \Phi(X = [x_1, x_2, \ldots, x_M]),$$
$$\text{s.t.} \begin{cases} K = \sum_{i=1}^{M} x_i, \\ x_i = 0 \text{ or } 1, \qquad i = 1, 2, \ldots, M, \end{cases} \tag{5}$$

where x_i represents the potential center of the disintegration circle. If the ith intersection point is selected as the location of the circle center, then $x_i = 1$, and $x_i = 0$ otherwise. We have defined $\Phi(X)$ as the metric to measure the disintegration effect, and our goal is to find the optimal disintegration strategy to maximize $\Phi(X)$.

2. Measures for node feature

To explore the mechanism of the spatial network disintegration strategy based on region centrality, we adopt several metrics to investigate the features of network nodes.

(i) Impact measure, ξ: this metric is designed to measure the efficiency of the exchange of information on the network, and it can be used to carry out an accurate quantitative analysis of information flow. The average efficiency of the network is defined as

$$\mathrm{eff} = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}, \tag{6}$$

where d_{ij} is the shortest path length that contains the least number of links between two nodes v_i and v_j. The impact measure that defines the network disintegration strategy is

$$\xi = \frac{\mathrm{eff}_G - \mathrm{eff}_{\hat{G}}}{r}, \tag{7}$$

where eff_G and $\mathrm{eff}_{\hat{G}}$ are the average efficiencies of the initial network G and the remaining network $\hat{G}$, respectively. The higher the value of eff, the better is the disintegration effect of the corresponding method.

(ii) Average number of removed nodes, δ is

$$\delta = \frac{n}{K}, \tag{8}$$

where n is the total number of nodes removed from the network and K is the number of disintegration circles.

(iii) Average degree of removed nodes, σ: define the state vector of the node as $C = [c_1, c_2, \ldots, c_N]$, where $c_i = 1$ if $v_i \in \hat{V}$, and $c_i = 0$ otherwise. k_i is the degree of node v_i. The average degree of removed nodes is then defined as follows:

$$\sigma = \frac{1}{n} \sum_{i=1}^{N} c_i k_i. \tag{9}$$

(iv) Deviation of the degree of removed nodes from the average degree, Θ,

$$\Theta = \frac{1}{n} \sum_{i=1}^{N} c_i |k_i - \langle k \rangle|, \tag{10}$$

where $\langle k \rangle$ is the average degree. Θ measures the deviation between the degree of the removed nodes and the average degree of the network. The smaller the value of Θ, the closer is the degree of the removed nodes to the average degree.

III. DISINTEGRATION STRATEGY FOR SPATIAL NETWORKS BASED ON REGION CENTRALITY

We divide the spatial coordinate system into $w \times w$ grids, and the problem is to find the optimal locations of K disintegration circles in the coordinate system to maximize $\Phi(X)$. The degree-based strategy is the most widely used network disintegration method. We can take the nodes with the high value of degree centrality as the centers of the disintegration circles. The betweenness also describes the importance of different nodes as a "bridge" in the network, i.e., the abilities of nodes or links to control the information of the network. A more traceable option is to remove the node with the highest degree or betweenness centrality with recalculation.

To obtain a better disintegration effect, we propose a solution based on region centrality. We define region centrality as the sum of centrality measures of all nodes or links in a specific area. Specifically, the region centrality is denoted as the sum of node degrees or the sum of links located within the circle (including the frontier of the circle) in spatial networks. Then, the spatial network is embedded in the normalized network coordinate system, and the parameter is set as $w = 100$ in this paper, which is equivalent to there being $M = w^2 = 10\,000$ options for the center position of the disintegration circle in the coordinate system.

Experimentally, we have found that the K disintegration circles would be concentrated in several network regions. Figure 2 visualizes the situation. The network in the figure is consistent with Fig. 1. It shows the case where two disintegration circles with different center positions contain the same node. The nodes v_4, v_6, v_7, and v_8 and the corresponding links will be removed. In this example, the sum of the node degrees in the two circles is the same, i.e., the disintegration efficiency of the two circles is equal to one. Similarly, for the disintegration strategy based on the number of links, this situation will also affect the disintegration effect.

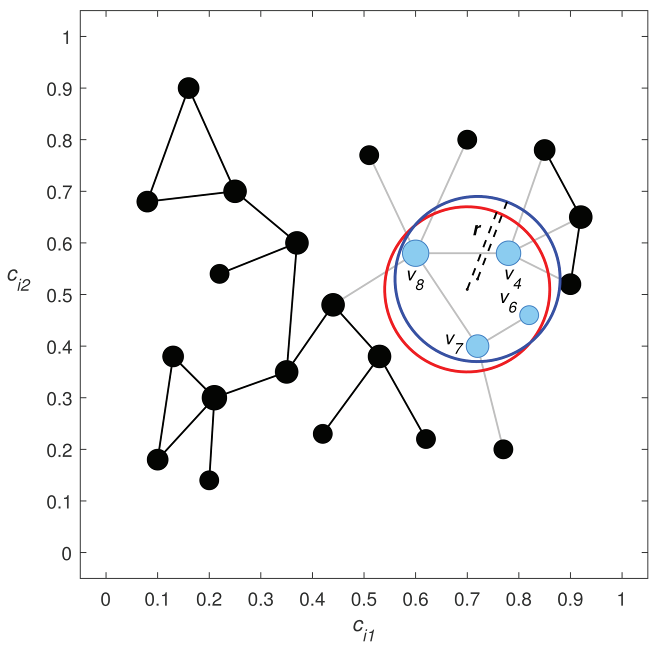

FIG. 2. Two disintegration circles with different center positions contain the same nodes. The blue circle and the red circle are different disintegration circles of radius r, and the sky-blue nodes are located in these two disintegration circles.

Therefore, inspired by the network attack strategy based on recalculated degree or betweenness at every removal step, we introduce a spatial network disintegration strategy with multiple disintegration circles based on the recalculated sum of node degrees (RSD) and the recalculated sum of links (RSE). First, we define a disintegration circle of radius r as C_r. Then, we consider the nodes that are located in the circle (including the boundary of the circle) and assign to circle C_r the sum of node degrees (SD) strength at level r according to the following equation:

$$\mathrm{SD}_r(l) = \sum_{v_j \in C_r(l)} k_j, \quad l = 1, 2, \ldots, M, \tag{11}$$

where $C_r(l)$ represents the lth disintegration circle of radius r, and k_j is the degree of node v_j located in the circle $C_r(l)$.

Likewise, we consider the links that are located in the circle (including links that are tangent to the circle) and assign to circle C_r the sum of links (SE) strength at level r according to the following equation:

$$\mathrm{SE}_r(l) = \sum_{e_j \in C_r(l)} e_j, \quad l = 1, 2, \ldots, M. \tag{12}$$

If the link of the network lies in the disintegration circle, then we set $e_j = 1$; otherwise, $e_j = 0$.

The steps of the spatial network disintegration strategy based on region centrality are as follows:

Step 1: In the beginning, all nodes in the network are present: $v_j = 1$ and $e_j = 1$ for all j. Then, traverse the optional locations of $M = 10\,000$ circle centers in the coordinate system. For the RSD method, calculate the sum of the node degrees in each disintegration circle, $\mathrm{SD}_r(l)$. For the RSE method, calculate the number of network links included in each disintegration circle, $\mathrm{SE}_r(l)$.

Step 2: Arrange the sum of node degrees or the number of links in descending order. Then, remove all nodes in the disintegration circle $v_j \in C_r(l)$ with highest $\mathrm{SD}_r(l)$ and set $v_j = 0$, or remove all links in the circle $e_j \in C_r(l)$ with highest $\mathrm{SE}_r(l)$ and set $e_j = 0$. If there is more than one disintegration circle with the highest $\mathrm{SD}_r(l)$ or $\mathrm{SE}_r(l)$, then randomly select one of these circles and remove the nodes or links in that circle.

Step 3: Calculate the relative size of the LCC and the NC of the remaining network after removing these nodes or links. Go to the next step.

Step 4: Calculate the sum of the node degrees $\mathrm{SD}_r(l^*)$ or the number of links $\mathrm{SE}_r(l^*)$ corresponding to each candidate disintegration circle of the remaining network, and then remove the nodes or links in the circle with highest $\mathrm{SD}_r(l)$ or $\mathrm{SE}_r(l)$.

Step 5: Go back to step 3. Repeat the procedure until K disintegration circles are found or the upper limit of the cost constraint is reached.

IV. APPLICATIONS TO SYNTHETIC NETWORKS

In this section, to demonstrate the applicability of the proposed spatial network disintegration strategy based on region centrality, we first apply it to some synthetic networks: the Erdös–Rényi (ER) model of random networks,[39] the Newman–Watts (NW) model of small-world networks,[40] and the Barabási–Albert (BA) model of scale-free networks.[13] Although random network models cannot accurately describe most real-world networks, such models can provide us with an essential reference when discussing the properties of real networks. Moreover, the NW model extension to the ER model successfully explains the coexistence of a high clustering coefficient and a low average geodesic length (small-world behavior) that is more in line with the characteristics of a real-world network. The construction of the ER and NW models is reflected in how links are placed between a fixed number of nodes. In contrast to these two models, the essence of the BA model is the inseparability of the structure and evolution of the network.

Many research works have studied the problem of network spatialization, which aims to transform the network into a map.[41,42] We generate spatial networks with geographical characteristics by randomly assigning coordinates to network nodes for ease of analysis. In the ER model, we fixed the number of nodes to $N = 200$ and assigned each node a horizontal and vertical coordinate in the range [0,1], and then map the nodes to a rectangular coordinate system with the coordinate range [0,1]. The nodes of the NW network and BA network mentioned later are mapped to the normalized network coordinate system in the same way. Subsequently, with a probability of 0.04, we link two randomly selected nodes with an link, and we perform this process for all node pairs of $N(N-1)/2$ to create a geographically characterized ER spatial random network. During the generation of the model, multiple links between two nodes are not allowed. Since the Watts–Strogatz (WS) model can produce isolated nodes, we adopt the NW model, in which only shortcuts are added and no links are removed from the original network. First, we create a ring over $N = 200$ nodes, and we then connect each node in the ring with its four nearest neighbors. Then, for each link (u, v) in the network, with a probability of 0.5, we add a new link (u, w) with a randomly chosen existing node w. Finally, we embed all nodes in the normalized network coordinate system. As for the BA model, a network of $N = 200$ nodes is grown by attaching new nodes, each with three links that are preferentially attached to existing nodes with a high degree. Similarly, nodes in the network are normalized.

We first show in Fig. 3, plots of the destructive effect of different network disintegration methods on the three synthetic networks as a function of the number of disintegration circles. We compare the spatial network disintegration strategy based on region centrality (using the recalculated sum of node degrees and recalculated sum of links, RSD and RSE) with five other methods: degree-core removal (DC), betweenness-core removal (BC), eigenvector-core removal (EC), recalculated-degree removal (RD), and recalculated-betweenness removal (RB). The basic idea of eigenvector centrality is that the node centrality is a function of the adjacent nodes' centrality. For DC, BC, and EC strategies, we select the nodes' coordinates in the descending order of three metrics in the network as the center locations of the disintegration circles and then remove all nodes in the corresponding circle one by one starting from the node with the highest degree, betweenness, or eigenvector centrality. Simultaneously, the RD and RB methods use the recalculated degree and betweenness centrality at every step to determine the circle's position and then the nodes located in the circle are removed circle by circle. For the RSD and RSE strategy, we put the center of the disintegration circle at the intersection of the 1000×1000 grid, and

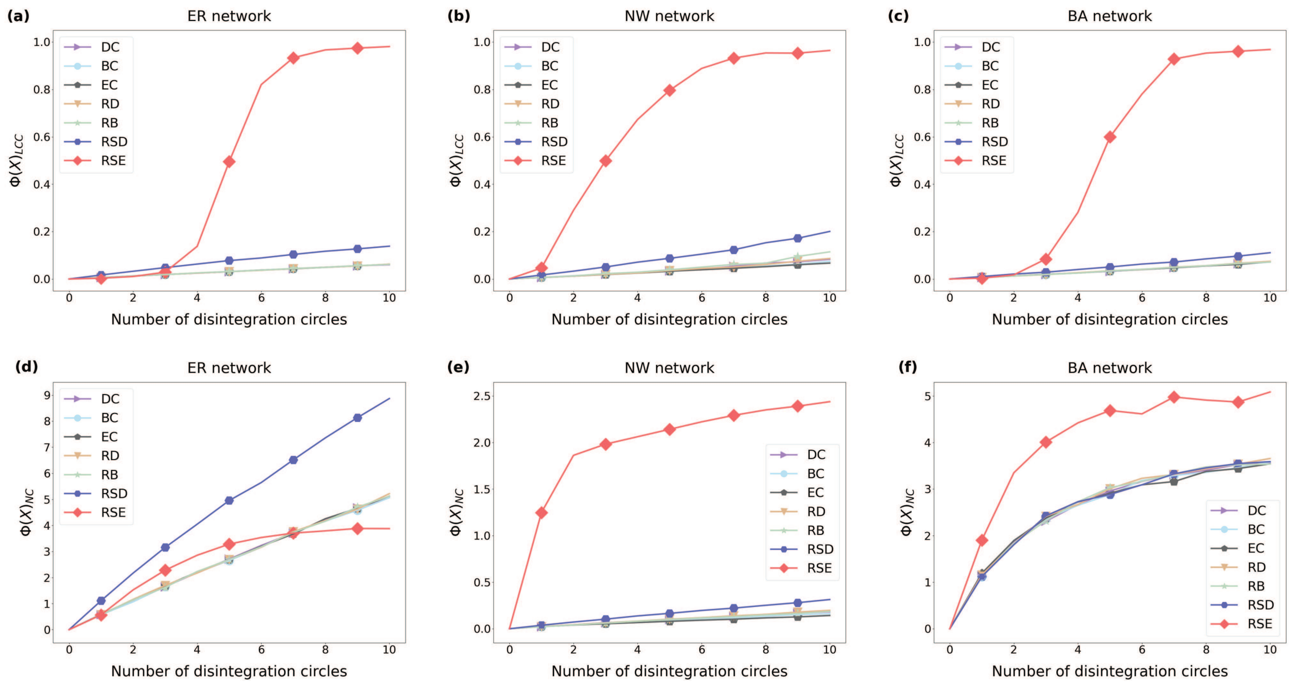

FIG. 3. Evaluation of different network disintegration methods on ER [(a) and (d)], NW [(b) and (e)], and BA [(c) and (f)] synthetic networks, for different numbers of disintegration circles. The methods are degree-core removal (DC, violet lines), betweenness-core removal (BC, sky-blue lines), eigenvector-core removal (EC, black lines), recalculated-degree removal (RD, brown lines), recalculated-betweenness removal (RB, dark-green lines), recalculated sum of node degrees (RSD, blue lines), and recalculated sum of links (RSE, red lines). In (a)–(c), the destructive effect on the network is measured by the function $\Phi(X)_{\mathrm{LCC}}$, representing the deterioration in the largest connected component. In (d)–(f), the destructive effect is measured by the function $\Phi(X)_{\mathrm{NC}}$, representing the deterioration in natural connectivity.

then remove the nodes or links within the disintegration circle with the highest $SD_r(l)$ or $SE_r(l)$ at every step. We fix the circle radius to $r = 0.04$. As can be seen from Fig. 3, our proposed strategy has a much better destructive effect on the synthetic networks for different numbers of disintegration circles than the other five methods. In particular, the destructive effect of the method based on the RSE dramatically exceeds those of the other methods. It also can be seen that when the destructive effect on the network is measured in terms of the LCC, if the number of circles satisfies $K \geqslant 7$, then for the RSE method, the corresponding $\Phi(X)_{\mathrm{LCC}} \to 1$ indicates that the largest connected component vanishes, i.e., $LCC = 0$, which also means that the network has been completely disintegrated. Furthermore, we show in Fig. 4 the destructive effect of the seven methods on the three synthetic networks as a function of the radius r. We fix the number of circles to $K = 10$. It can again be seen that our strategy consistently outperforms the other methods in different application scenarios, with the RSE method having the outstanding performance.

V. APPLICATIONS TO REAL-WORLD NETWORKS

A. Data description

Since synthetic networks cannot fully describe the various properties of real-world networks, to verify the applicability of the proposed network disintegration strategy to realistic scenarios, we evaluate it on two real-world networks of different types: the American air network (http://vlado.fmf.uni-lj.si/pub/networks/data/) and the Minnesota road network (http://networkrepository.com/index.php). The networks are constructed as follows. In the American air network, nodes represent airports, and links represent the routes between airports. The Minnesota road network is composed of town nodes and links representing roads between towns. Moreover, the longitude and latitude coordinates of the airports and the towns are normalized by Eq. (1), and these normalized node coordinates are embedded in the rectangular coordinate system in the range $[0, 1]$. Table I shows the basic topological characteristics of the two real-world networks.

We apply the spatial network disintegration strategy to these real-world networks, not with the aim of attacking them, but rather to use the strategy to find the vital nodes or links in a network more efficiently. In other words, an attack on these specific sets of nodes or links may cause the network structure to disintegrate into many small isolated subcomponents. On the other hand, the strategy can also provide theoretical support for strengthening some nodes, links, or areas of the network.

B. Results

To more clearly illustrate the relative locations of the disintegration circles in the spatial networks, we visualize these circles

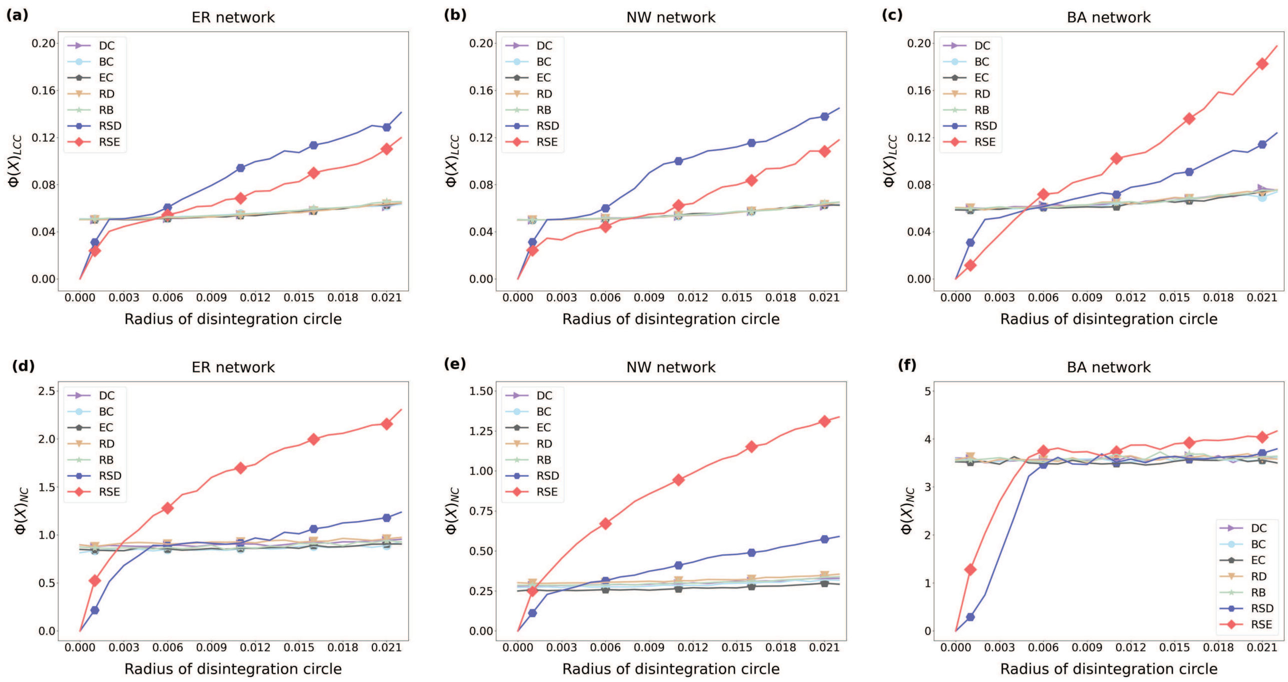

FIG. 4. Evaluation of different network disintegration methods on ER [(a) and (d)], NW [(b) and (e)], and BA [(c) and (f)] synthetic networks, for different values of the radius r. The methods are degree-core removal (DC, violet lines), betweenness-core removal (BC, sky-blue lines), eigenvector-core removal (EC, black lines), recalculated-degree removal (RD, brown lines), recalculated-betweenness removal (RB, dark-green lines), recalculated sum of node degrees (RSD, blue lines), and recalculated sum of links (RSE, red lines). In (a)–(c), the destructive effect on the network is measured by the function $\Phi(X)_{\rm LCC}$, representing the deterioration in the largest connected component. In (d)–(f), the destructive effect is measured by the function $\Phi(X)_{\rm NC}$, representing the deterioration in natural connectivity.

in Fig. 5. We compare the RSD and RSE disintegration strategy with five other methods: degree-core removal, betweenness-core removal, eigenvector-core removal, collective influence-core removal, and non-backtracking-core removal (NBC).[43] The CI is a metric for identifying influential spreaders in the network and quantify the propagation ability. Non-backtracking centrality is a spectral centrality metric based on the non-backtracking matrix, which solves the problem that most of the centrality's weight of eigenvector centrality concentrates on a small number of nodes. The node with the greatest value determined by the five strategies is taken as the center of the disintegration circle. For the CI strategy, we take a ball of unit radius around every node. We fix the number of circles to $K = 10$. For both the American air network and the Minnesota road network, the positions of the circles determined by the different disintegration strategies overlap with each other, and so the distributions of all the circles cannot be shown on the figure. As can be seen, since the circle center determined by the DC, BC, EC, CI, and NBC methods is located at the node, the distribution of the circle in the network will not change significantly with increasing radius. Our proposed strategy based on region centrality is more likely to find circles covering a greater number of nodes or links than the five methods based on centrality metrics. With increasing radius, the circles identified by our strategy are more clustered in areas of the network where there are more nodes or links. A more interesting phenomenon that we can observe from Fig. 5 is that the red circles and blue circles in the network are more widely distributed, and most of the circles are located in areas with a higher density of nodes or links. By contrast, the remaining circles cover areas with suboptimal network connectivity. Although the distribution of circles determined by the BC method is also not concentrated, the number of circles located in areas with higher network density is relatively small, and these are the core areas of the network.

TABLE I. Basic statistics for the two networks under consideration, where N and W are the numbers of nodes and links, d and $\langle k \rangle$ are the density and average degree of the network, D is the diameter, and C is the average clustering coefficient.

Network	N	W	d	$\langle k \rangle$	D	C
American air	332	2126	0.038 69	12	6	0.6252
Minnesota road	2642	3303	0.000 95	2	99	0.0160

In Fig. 6, we show the disintegration effect of the seven methods on the American air network and Minnesota road network as a function of the radius r. It can be seen that the performance of the disintegration strategy based on region centrality is significantly better than those of the other methods on two real-world networks for both measure functions and for different radii, especially for the Minnesota road network. These results show that our strategy

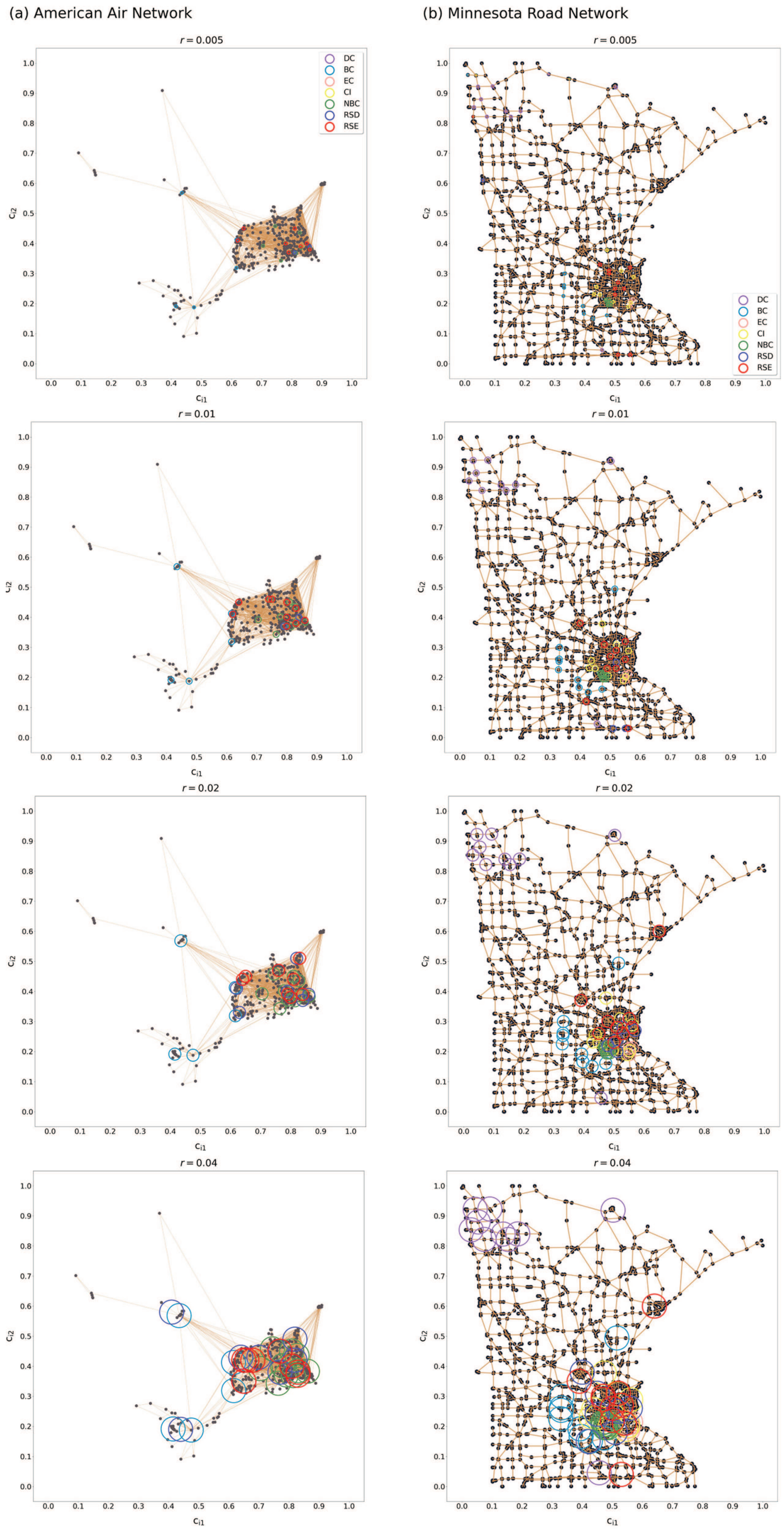

FIG. 5. Location of disintegration circles of radii $r = 0.005$, 0.01, 0.02, and $r = 0.04$ in (a) the American air network and (b) the Minnesota road network for the seven methods: degree-core removal (violet circles), betweenness-core removal (sky-blue circles), eigenvector-core removal (pink circles), collective influence-core removal (yellow circles), non-backtracking-core removal (dark-green circles), recalculated sum of node degrees (blue circles), and recalculated sum of links (red circles).

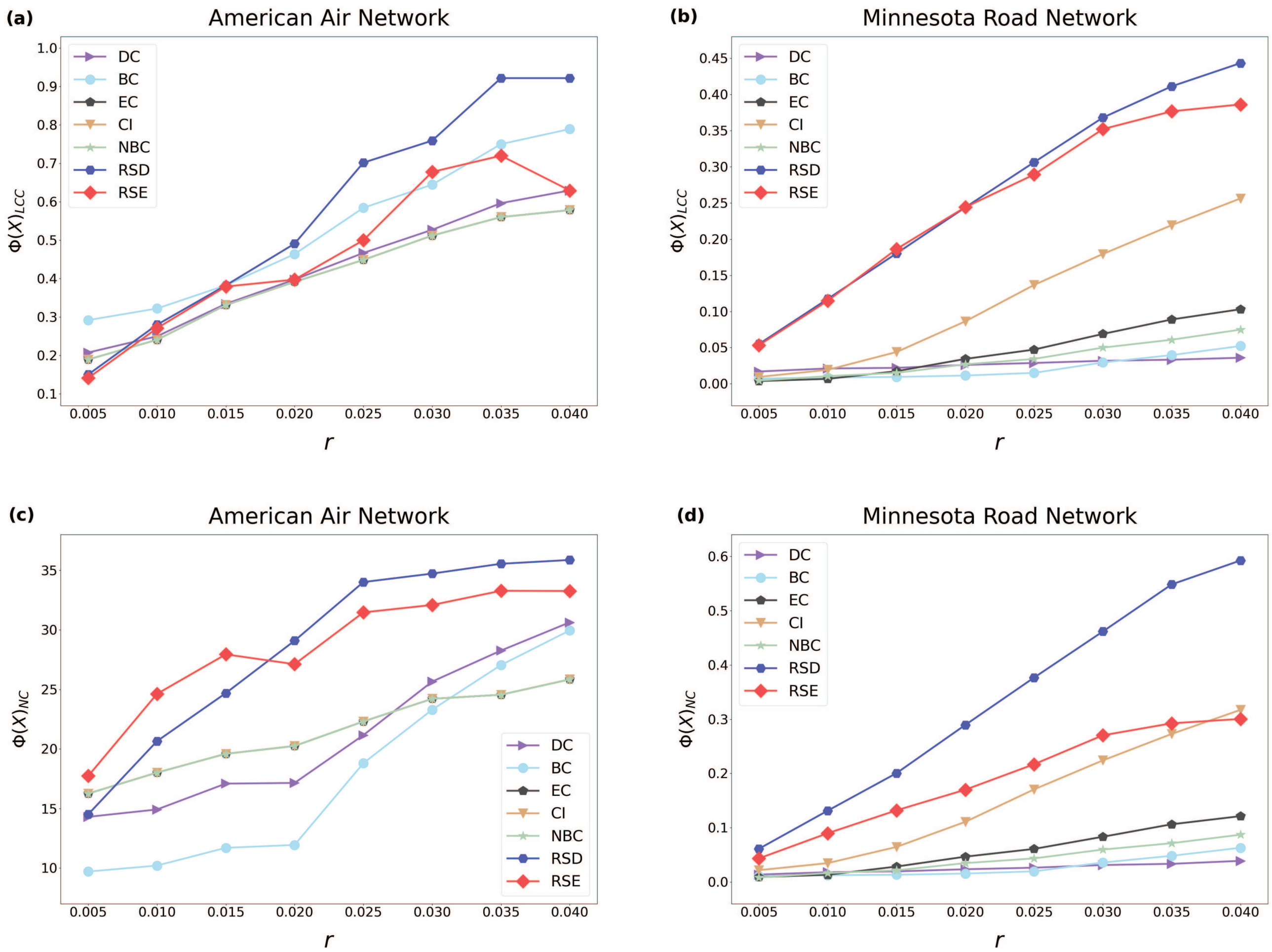

FIG. 6. Evaluation of different network disintegration methods on the American air network [(a) and (c)] and the Minnesota road network [(b) and (d)] for different values of the radius r. The methods are degree-core removal (DC, violet lines), betweenness-core removal (BC, sky-blue lines), eigenvector-core removal (EC, black lines), collective influence-core removal (CI, brown lines), non-backtracking-core removal (NBC, dark-green lines), recalculated sum of node degrees (RSD, blue lines), and recalculated sum of links (RSE, red lines). In (a) and (b), the destructive effect on the network is measured by the function $\Phi(X)_{\mathrm{LCC}}$, representing the deterioration in the largest connected component. In (c) and (d), the destructive effect is measured by the function $\Phi(X)_{\mathrm{NC}}$, representing the deterioration in natural connectivity.

can achieve better disintegration effects on spatial networks than conventional methods. Table II shows the connected component characteristics of the remaining networks after the two real-world networks are attacked. It can be seen that the RSD strategy tends to break the network into more connected components with a size not larger than the RSE strategy. In terms of the effects of network disintegration, the RSD strategy is superior to the RSE strategy for two considered real-world networks.

To further explore the disintegration mechanism of our strategy and the resulting network changes, we applied the four measures introduced in Sec. II B 2 to evaluate the features of the nodes removed from the network with different strategies under different disintegration circle radius, as shown in Fig. 7. It can be seen from the changing pattern of the histograms that, except for the average number of removed nodes, all measures exhibit a decreasing trend with increasing radius, which can be interpreted as meaning that a larger radius of the disintegration circle can cover more nodes or links, particularly for the implementation of the RSD and RSE strategies in the American air network. Although more nodes or links are removed using this strategy, the average degree of the removed nodes is smaller than with other methods, indicating that the location of the circles determined by this strategy will also include some nodes with low centrality. With regard to the impact measure, when our strategy is applied to the American air network, it outperforms other methods for large values of the radius, and its performance on the Minnesota road network is significantly better than that of other methods for all values of r considered, which indicates that the spread of information in the network is more hindered. The traffic between cities is severely affected in both the real air and road networks. To a certain extent, our strategy also provides a

TABLE II. Connected component characteristics of the remaining networks under consideration, where *NCC* and $C(Q)$ are the numbers of connected components and the number of nodes in the largest connected component, $\Phi(X)_{LCC}$ is the deterioration of the relative size of the largest connected component.

Network	Metric	RSD	RSE
American air	NCC	30	3
	$C(Q)$	26	123
	$\Phi(X)_{LCC}$	0.9217	0.6295
Minnesota road	NCC	11	3
	$C(Q)$	1469	1620
	$\Phi(X)_{LCC}$	0.4432	0.3861

reference for city nodes and traffic links that need to be protected, with these cities and the routes or roads between them corresponding to nodes or links located within the circle in the spatial network. At the same time, with our strategy, the deviation of the degree of removed nodes from the average degree is lower than for other methods. The smaller this deviation, the more likely is the circle determined by our strategy to cover those nodes close to the average degree of the network than to be concentrated on nodes with higher centrality.

VI. CONCLUSION AND DISCUSSION

Extensive numerical experiments on synthetic graphs and real-world networks have demonstrated that a disintegration strategy for spatial networks based on region centrality significantly outperforms conventional methods based on centrality and heuristic algorithms in terms of effectiveness and efficiency. This strategy seeks to find a set of nodes or links that allow a network to be disintegrated into many small components in a more efficient way. We define region centrality as the sum of centrality measures of all nodes or links in a specific area. This strategy comprehensively considers the topological properties and geographic structure of a spatial network. The disintegration circles determined by this strategy tend to cover those nodes close to the average degree of the network rather than being concentrated on the nodes with higher centrality, which naturally leads to a higher cost of disintegration. In addition, the distribution of circles in the network is relatively dispersed, and most of the circles are located in areas with a higher density of nodes or links, while the remaining circles cover areas with suboptimal network connectivity.

The performance of the proposed strategy can be further improved by considering other features of the nodes or links in the disintegration circle to more effectively destroy the network structure. It might even be possible to avoid direct use of the disintegration circle model, which is an exciting area for further investigation.

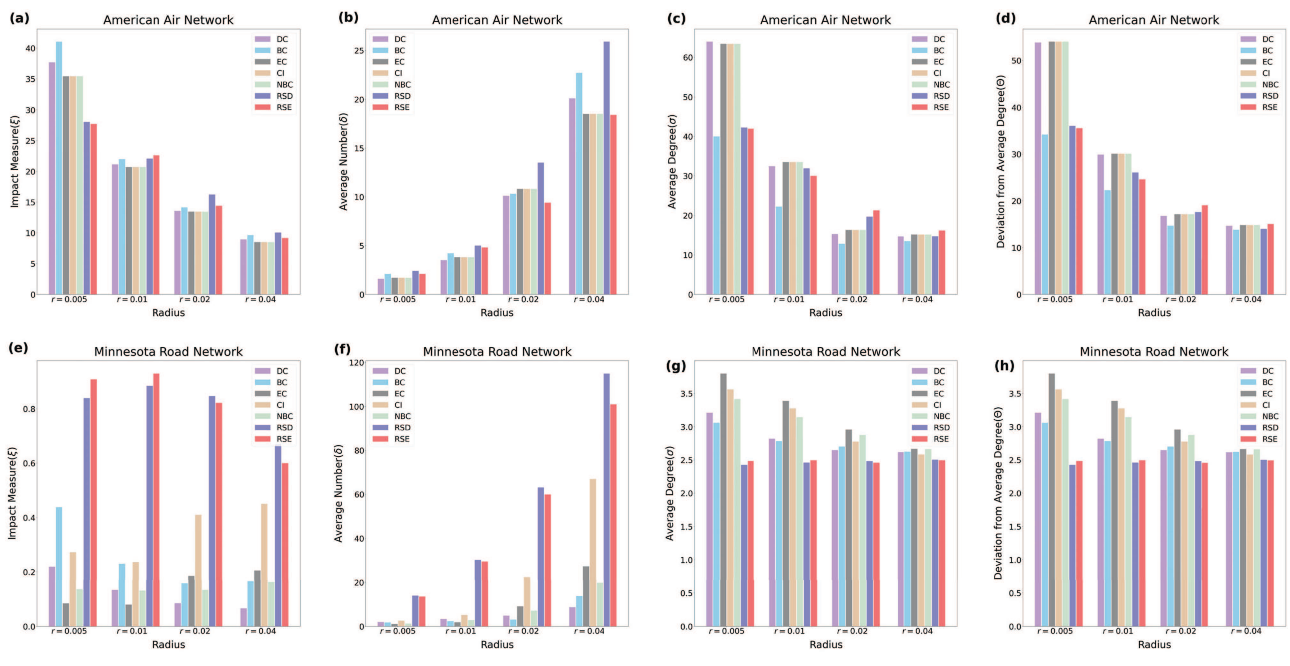

FIG. 7. Evaluation by four statistical measures of the residual networks of the American air network [(a)–(d)] and the Minnesota road network [(e)–(h)] after attack by different network disintegration methods for different values of the radius r. The methods are degree-core removal (DC, violet bars), betweenness-core removal (BC, sky-blue bars), eigenvector-core removal (EC, black bars), collective influence-core removal (CI, brown bars), non-backtracking-core removal (NBC, dark-green bars), recalculated sum of node degrees (RSD, blue bars), and recalculated sum of links (RSE, red bars). The measures used are the impact measure of disintegration in (a) and (e), the average number of removed nodes in (b) and (f), the average degree of removed nodes in (c) and (g), and the deviation of the degree of removed nodes from the average degree in (d) and (h).

Finally, directed and weighted networks are both currently active areas of research, and we hope to incorporate these types of spatial networks in future work.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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