Physica A 507 (2018) 435–442

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Efficient disintegration strategy in directed networks based on tabu search

Yang Yu, Ye Deng, Suo-Yi Tan, Jun Wu *

College of Systems Engineering, National University of Defense Technology, Changsha, Hunan 410073, PR China

HIGHLIGHTS

- An optimization model for disintegration strategy in directed networks is established.
- An efficient disintegration strategy based on tabu search is proposed.
- This strategy is more efficient than others based on degree and betweenness.
- The critical nodes in directed networks may not have large betweenness centrality.

ARTICLE INFO

Article history:
Received 22 June 2017
Received in revised form 5 March 2018
Available online 15 May 2018

Keywords:
Optimal disintegration strategy
Tabu search
Directed networks

ABSTRACT

The problem of network disintegration, which aims at identifying the critical nodes or edges whose removal will lead to a network collapse, has attracted much attention due to its wide applications. This paper focuses on the disintegration of directed networks. We propose a disintegration strategy based on tabu search. Experiments show that the disintegration effect of our strategy is obviously better than those of typical disintegration strategies based on local structural properties. Moreover, we find that the critical nodes identified to remove in directed networks are not those nodes with large degree or betweenness centrality that always are the crucial properties in undirected network.

1. Introduction

Complex networks describe a wide range of systems in the nature and society [1]. Examples include power grids, social networks, airline networks, urban road networks, terrorist networks, the Internet, etc. The study of complex network has become one of the most popular topics in interdisciplinary areas in the past decades. Due to its broad applications, studying on the robustness of complex networks has received growing attention and has become one of the central topics in the complex network research [2–7].

In the majority of cases, networks are beneficial, such as power grids and the Internet, whose function we want to preserve. Therefore, many studies have developed methods for enhancing the robustness of these beneficial networks [8–17]. In other situations by which this paper is motivated, we want to disintegrate some networks if they are harmful such as immunizing the population in social networks or suppressing the virus propagation in computer networks. Other examples of network disintegration include destabilizing terrorist networks [18], preventing financial contagion [19], controlling the rumor diffusion [20], and perturbing cancer networks [21]. The problem of network disintegration aims actually at identifying a set of critical nodes or edges for removal for the sake of collapsing a network, which has wide practical application in destroying network structure and controlling the information diffusion.

* Corresponding author.
E-mail address: junwu@nudt.edu.cn (J. Wu).

https://doi.org/10.1016/j.physa.2018.05.079

Many approaches for network disintegration have been proposed in the last decades [22,23]. Early disintegration strategies were related to certain structural metrics of nodes or edges. The most classical strategy is the degree-based strategy [2]. It is shown that scale-free networks with power-law degree distributions are extremely vulnerable to intentional attacks, in which nodes are removed sequentially by the decreasing order of their degrees. Considering that the degree is a local property of the node, the betweenness centrality is introduced as a new criterion [24,25]. Other global metrics were also used to identify the vital nodes for network disintegration [26], such as coreness [27] and subgraph centrality [28]. To achieve better disintegration effect, other methods were proposed [29–33].

Most previous researches on the problem of network disintegration have been limited to undirected networks. However, most real-world networks are actually directed, such as transportation networks [34] and terrorist networks [18]. Studying on the problem of disintegration in directed networks is of great significance, but few researches concentrated on this problem, especially with the view of global optimization. In this paper, we focus on the optimal disintegration strategy in directed networks. We will introduce tabu search to identify the critical nodes. Experiments show that the disintegration effect of optimal strategy is obviously better than the strategies guided by local properties of nodes.

The paper is organized as follows. In Section 2, we present the optimization model for disintegration strategy in directed networks. In Section 3, we investigate the solution based on tabu search for the optimization model. In Section 4, the experiments in various model networks are shown. And in Section 5, we present the conclusions and discussions.

2. Optimization model for disintegration strategy in directed networks

Consider networks represented by a simple directed graph $G = (V, E)$ having no loops and multiple edges, where V is the set of nodes and E is the set of edges. Let $N = |V|$ be the number of nodes and $W = |E|$ be the number of edges, respectively. Let $A(G) = (a_{ij})_{N\times N}$ be the adjacency matrix of G, where $a_{ij} = 1$ if there is a directed edges from v_i to v_j, and $a_{ij} = 0$ otherwise. Let $k_i^- = \sum_{j=1}^{N} a_{ji}$ and $k_i^+ = \sum_{j=1}^{N} a_{ij}$ be the in-degree and out-degree of node v_i, respectively. It is easy to obtain that

$$\sum_{i=1}^{N} k_i^- = \sum_{i=1}^{N} k_i^+ = W. \tag{1}$$

We only consider disintegration approaches against nodes in this study and assume that the attached edges are removed if one node is removed. Denote by $\hat{V} \subseteq V$ the set of nodes that are removed and denote by $\hat{G} = (V - \hat{V}, \hat{E})$ the network after the node removal. Denote by $n = |\hat{V}|$ the disintegration strength parameter. We define a disintegration strategy as $\hat{X} = [x_1, x_2, \ldots, x_N]$, where $x_i = 0$ if $v_i \in \hat{V}$, otherwise $x_i = 1$. Then we obtain

$$n = N - \sum_{i=1}^{N} x_i. \tag{2}$$

The measure function of network performance is denoted by $\Gamma(G)$. We assume that if $G_1 = (V_1, E_1)$ is a subgraph of $G_2 = (V_2, E_2)$, i.e. if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$, then $\Gamma(G_1) \le \Gamma(G_2)$. This monotonicity assumption ensures that the network performance reduces with the process of network disintegration. We define the effect of disintegration strategy as the degradation of network performance after node removal $\Phi(\hat{X}) = \Gamma(G) - \Gamma(\hat{G}) \ge 0$. Our goal is to find the optimal disintegration strategy $\hat{X}^*$, which maximizes the disintegration effect $\Phi(\hat{X})$. Therefore, we obtain the optimization model for disintegration strategy as follows

$$\begin{aligned} &\max\ \Phi(\hat{X} = [x_1, x_2, \ldots, x_N]) \\ &\text{s.t.} \begin{cases} \displaystyle\sum_{i=1}^{N} x_i = N - n \\ x_i = 0 \text{ or } 1, i = 1, 2, \ldots, N \end{cases} . \end{aligned} \tag{3}$$

3. Solution based on tabu search

As the objective function Φ generally has no explicit form, we cannot solve the optimization model in Eq. (3) using traditional integer programming techniques. Actually, it can be seen as a combinatorial optimization problem. For a network with N nodes, there are C_N^n ways to pick up n nodes for removal, which grows dramatically with N and n. For example, for $N = 100$ and $n = 10$, the total number of alternative strategies will be astronomically large, corresponding to $C_{100}^{10} = 100!/(10! \times 90!) \approx 1.73 \times 10^{13}$. Therefore, traversing all possible solutions is almost impossible if the network size is large. Here, we consider solving this problem using tabu search (TS).

For most cases, tabu search is a meta-heuristic search algorithm and used as an efficient tool for solving global optimization problems [35]. It employs local search methods which take the improved one among immediate neighbors of the current solution as the next solution iteratively until any improved solution cannot be found. By introducing mutation operation, the ability of escaping the local optimum is strengthened and the ability of finding global optimum is enhanced. TS has five important parts to ensure its fundamental function: move mechanism, prohibition rule, tabu list, aspiration criterion, termination criterion. Move mechanism represents the process of aiming to the best solution. The basic principle of move mechanism is to obtain an initial potential solution, and check its immediate neighbors to choose the best one for next step iteratively. The immediate neighbors of present solution are generated by a conduction "swap". A worsening move can be accepted for avoiding falling into in sub-optimal regions during the moving process. The prohibition rule is established to forbid the search coming back to previously visited solutions. This rule is realized based on tabu list, a structure for recording the previous chosen swap in each moves. Moving to a swap in tabu list is forbidden unless the corresponding solution can satisfy the aspiration criterion. The aspiration criterion is set to break the prohibition rule in some particular situations in order to avoid the missing of good solution and stimulate the occurrence of optimal solution. Finally, the termination criterion is the rules for stopping the process and is generally set according to the upper limit of iteration times.

Actually, running only one time for tabu search algorithm is hard to find the optimal solution. Therefore, the operation of restarting is necessary for a better solution. However, the computation time is also an important factor for evaluating the performance of algorithm. Over many times of restarting can make the total cost of computation time be pretty large especially when the time cost in each round is large. Hence, we set a parameter of the maximum total number of iterations rather than a certain number of restarting to control the times of restarting. After each round of searching, the program will restart until the total number of iterations reaches the maximum.

We provide a description of the solution for optimal disintegration strategy based on tabu search in more details. As shown in optimization model, a disintegration strategy can be regarded as a possible solution in TS. The aim of this process is to find the optimal solution maximizing the value of object function (the disintegration effect). The initial solution $\hat{X}_0$ can be produced by removing n nodes randomly according to disintegration strength parameter n. Then let the initial solution be the current solution. The current solution at each step is denoted by $\hat{X}_{cur}$, then let $N(\hat{X}_{cur})$ denote the neighbors of the current solution. Each neighbor of current solution is obtained by exchanging the states ("0" or "1") of a pair of nodes randomly, which is denoted as a swap S. Since the number of possible neighbors is large, we choose n_{can} neighbors randomly as candidate solutions. One of the candidate solutions will be chosen as "current solution" in the next step if it has the best disintegration effect but not belongs to tabu list, or satisfies the aspiration criterion. And the swap producing this "best" candidate solution is denoted as the "best" swap at this step. If candidates have the same performance, the solution can be chosen randomly. Disintegration effect of $\hat{X}_i$ is measured by function $\Phi(\hat{X}_i)$, which is defined in optimization model. The best-known solution is indicated by $\hat{X}_{opt}$, and the corresponding value of disintegration effect is denoted by $\Phi(\hat{X}_{opt})$.

Tabu list, a structure whose length is L, is denoted as T_{list} to record the previous best swaps. Tabu list is empty at first, and the "best" swap at each step is recorded into tabu list once conducted. When we choose the best swap from the candidate solutions, the swap belonging to tabu list should be abandoned. The length of tabu list L determines the number of iterations that the "best" swaps cannot be chosen. The swaps in tabu list for more than L iterations can be released. However, the tabu list should be emptied if the $\hat{X}_{opt}$ is updated to a better one, because the better $\hat{X}_{opt}$ indicates the algorithm does not fall into the sub-optimal solution and the previous good swaps can be accepted. Aspiration criterion is that if a candidate swap is tabued but its corresponding solution has $\Phi(\hat{X}_{can})$ better than the current $\Phi(\hat{X}_{opt})$, this swap would be released from the tabu list and conducted. Termination criterion is set to stop a round of process. The current number of iteration is denoted by n_{iter} while the best-known solution is not updated. The value of n_{iter} would be set to 0 if $\hat{X}_{opt}$ is updated. When the current number of iterations n_{iter} arrives at the maximum number of iterations n_{max} set previously without improved solution, this process ends. Finally, the temporary final optimal solution is the current best-known solution.

The above process can be seen as a round of iterations or a round of searching, and the number of iterations in a round is uncertain. We denote the maximum total number of iterations as T_{max} and the total number of iterations in all past rounds as T_{iter}. When a round of iterations is completed but T_{iter} is less than T_{max}, the program will produce a new initial solution randomly and restart a new round of searching. The program can eventually stop if T_{iter} exceeds T_{max}. The best-known solution in all rounds is the final result.

The detailed steps for solution based on tabu search algorithm is described as follows:

- Step 1. Set up the length of tabu list L, the maximum iteration number n_{max}, the maximum total iteration number T_{max}, the number of candidate strategies n_{can};
- Step 2. Generate initial strategy $\hat{X}_0$ by removing n nodes randomly, i.e. set $x_i = 0$ of $\hat{X}_0 = [x_1, x_2, \ldots, x_N]$ when v_i is removed;
- Step 3. Let the current best-known solution $\hat{X}_{opt} = \hat{X}_0$, and calculate $\Phi(\hat{X}_{opt})$;
- Step 4. Initialize $T_{iter} = 0$;
- Step 5. Initialize $n_{iter} = 0$, $T_{list} = \text{NULL}$ and $\hat{X}_{cur} = \hat{X}_0$;
- Step 6. Produce n_{can} candidate strategies $\hat{X}_i \in N(\hat{X}_{cur})$ through swap S_i respectively;
- Step 7. Find out $\hat{X}_k$, s.t. $\Phi(\hat{X}_k) = \max \Phi(\hat{X}_i)$;
- Step 8. If $S_k \notin T_{list}$ or $\Phi(\hat{X}_k) > \Phi(\hat{X}_{opt})$, then $\hat{X}_{cur} = \hat{X}_k$; otherwise, find out $\hat{X}_k$, s.t. $\Phi(\hat{X}_k) = \max \Phi(\hat{X}_i)$ that $S_k \notin T_{list}$, then $\hat{X}_{cur} = \hat{X}_k$;
- Step 9. Remove element added into T_{list} for over L times of iterations, then add swap S_k to T_{list}, update T_{list};
- Step 10. If $\Phi(\hat{X}_{cur}) > \Phi(\hat{X}_{opt})$, then $\Phi(\hat{X}_{opt}) = \Phi(\hat{X}_{cur})$, $\hat{X}_{opt} = \hat{X}_{cur}$, $T_{list} = \text{NULL}$ and $n_{iter} = 0$; else $n_{iter} = n_{iter} + 1$;
- Step 11. $T_{iter} = T_{iter} + 1$. If $T_{iter} < T_{max}$, then turn to Step 12; else exit;
- Step 12. If $n_{iter} < n_{max}$, then turn to Step 6; else, generate a new initial strategy $\hat{X}_0$ and turn to Step 5.

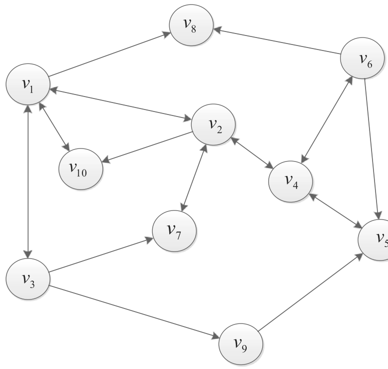


Fig. 1. An example network with 10 nodes and 21 directed edges.

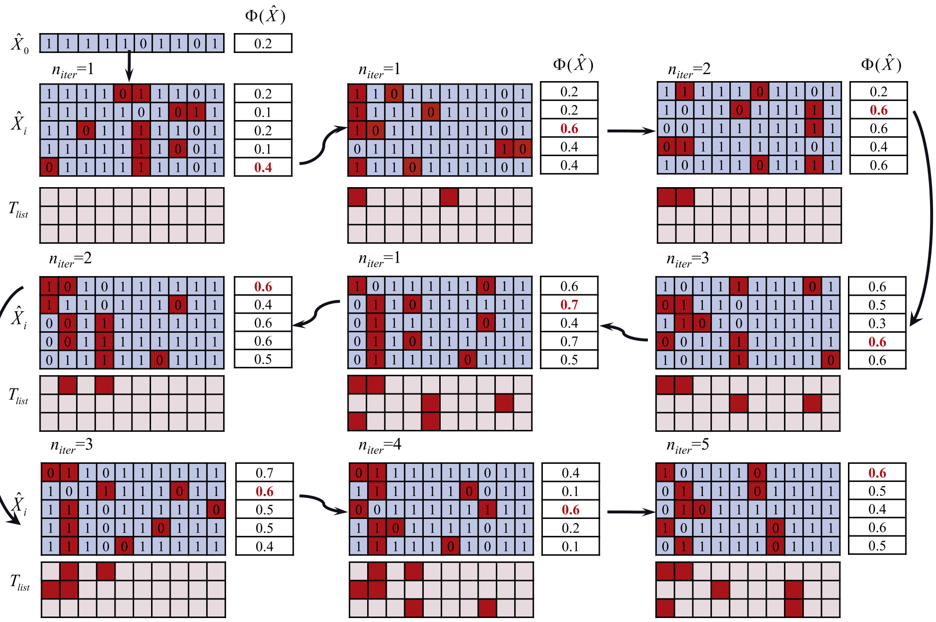


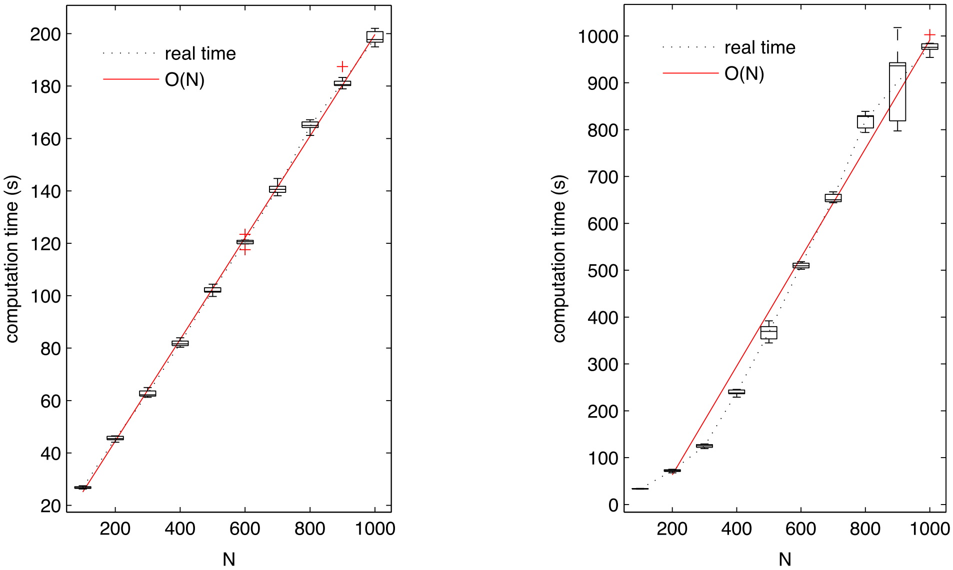
Fig. 2. Illustration for the disintegration strategy based on tabu search algorithm.

Now we use a simple example to illustrate the disintegration strategy based on tabu search in one round. There is a network with 10 nodes and 21 directed edges shown in Fig. 1. In this experiment, we use the largest strongly connected component (LSCC) as the measure function of network performance $\Gamma(G)$. LSCC is defined as the proportion of nodes in largest subgraph where any two nodes can arrive at each other by directed edges. The effect of disintegration strategy is obtained by $\Phi(\hat{X}) = \Gamma(G) - \Gamma(\hat{G}) \geq 0$. Here, the value of disintegration effect is the degradation of LSCC after removal. The initial LSCC is 0.9. We assume that the number of candidate solutions n_{can} is 5, the maximal times of iteration n_{max} is 5, the length of tabu list L is 3, and the disintegration strength parameter n is 2. The whole process for obtaining optimal disintegration strategy is illustrated in Fig. 2.

We first generate an initial solution randomly

$$\hat{X}_0 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}, \tag{4}$$

which is indicated by the top row of cells in Fig. 2. For the initial solution, node 6 and node 9 are removed whose state are denoted by "0", LSCC $= 0.7$ and $\Phi = 0.2$. Then, we generate 5 candidate solutions by swapping the states between a removed node and a reserved node randomly for 5 times. Each row of blue cells in Fig. 2 represents a candidate solution. Cells marked in red donate swapped nodes inherited from the current solution. Next, the current solution moves to the best solution among available candidate solutions step by step until the best-known solution remains unchanged during 5 steps. The right column presents the disintegration effect of each candidate solution respectively, and the value of objective



(a) CM(N, $m = 3$, $\lambda = 2.5$, $q = 0.5$). (b) ER(N, $p = lnN/N$, $q = 0.5$).

Fig. 3. The computation time of the disintegration strategy based on TS versus the network size N in directed scale-free networks $CM(N, m, \lambda, q)$ (a) and directed random networks $ER(N, p, q)$ (b), where the disintegration strength $n = 0.1 \times N$. The box plots are drawn based on over 100 independent realizations of network generation. The fitting curves based on $O(N)$ are also shown as reference lines. The calculation time is the total running time including the restarting operations.

function with best effect is marked in red. Tabu list is represented by light red cells below each group cells of candidate solutions, and every row represents a tabu element. In tabu list, cells in red represent selected swaps at last 3 steps, which cannot be conducted temporarily. Finally, we obtain the optimal disintegration strategy which removes the node 1 and node 4.

The computation cost of the disintegration strategy based on tabu search is influenced by several factors, including the number of nodes N, the number of edges W and the attack strength n. We show the computation time as a function of network size in Fig. 3. We find that the time complexity of the disintegration strategy based on TS is approximately $O(N)$. Furthermore, the band inside the box and the length of whiskers are both short and there are few outliers, which indicate that the calculation performance of this algorithm is steady. Therefore, the computation cost of this method based on tabu search is acceptable. It is worth pointing out that we have repeated our experiments with various parameter values and observe the similar results.

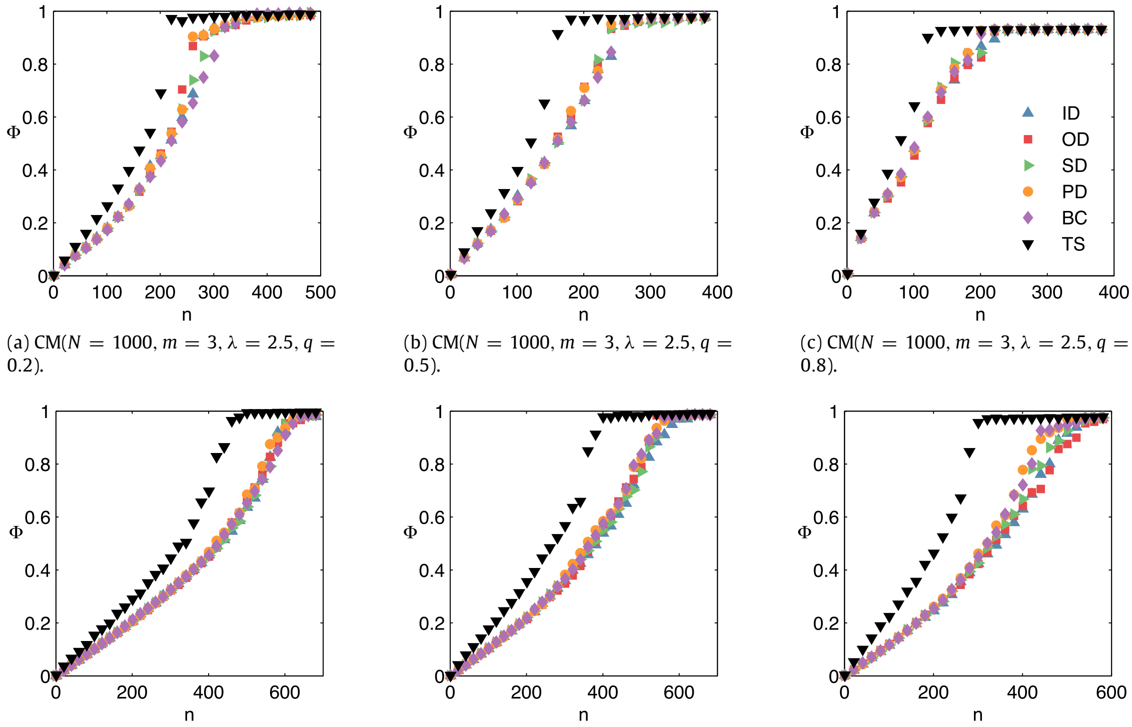
4. Experimental analysis

In this section, we will analyze the optimal disintegration strategies based on the optimization model introduced above. Due to the ubiquity in the real-world, we focus on directed scale-free networks and directed random networks in this study. We first generate an undirected network and then set a certain proportion q of two-way edges as one-way edges randomly for generating a directed network. The scale-free network with power-law degree distributions $p(k) = (\lambda - 1)m^{\lambda-1}k^{-\lambda}$ is generated by the configuration model [1]. Therefore, the directed scale-free network model is denoted by $CM(N, m, \lambda, q)$. The undirected random network can be generated by a pair of parameters, the number of nodes N and the possibility of linking between any two nodes p, so the directed random network model can be indicated by $ER(N, p, q)$.

The parameters for solving the optimization model for disintegration strategy based on tabu search is set as $L = 10$, $T_{max} = 1000$, $n_{max} = 100$ and $n_{can} = 100$. These experiments are conducted on a PC (Intel Core i5-4210U CPU@2.4 GHz, 4 GB RAM) with MATLAB version R2014a.

To demonstrate the efficiency of our strategy based on tabu search (TS), we compare it with five typical disintegration strategies based on local structural properties: removing nodes in decreasing order of the in-degree (ID), the out-degree (OD), the sum of in-degree and out-degree (SD), the product of in-degree and out-degree (PD) and the betweenness centrality (BC). We use the size of LSCC as the measure function of network performance $\Gamma(G)$. The disintegration effects $\Phi(\hat{X})$ as a function of the disintegration strength parameter n with various disintegration strategies in directed scale-free networks and directed random networks are shown in Fig. 4. We find that, given the disintegration strength parameter, the disintegration effect of TS is much better than other disintegration strategies.

To compare the effects of various disintegration strategies in depth, we also show the critical removal fraction of nodes f_c [2] as a function of the proportion of one-way edges q in Fig. 5. This measure considers statistically how the removal of



(a) CM($N = 1000, m = 3, \lambda = 2.5, q = 0.2$).
(b) CM($N = 1000, m = 3, \lambda = 2.5, q = 0.5$).
(c) CM($N = 1000, m = 3, \lambda = 2.5, q = 0.8$).
(d) ER($N = 1000, p = 0.008, q = 0.2$).
(e) ER($N = 1000, p = 0.008, q = 0.5$).
(f) ER($N = 1000, p = 0.008, q = 0.8$).

Fig. 4. The disintegration effect Φ versus the disintegration strength parameter n with various disintegration strategies in random directed scale-free networks $CM(N, m, \lambda, q)$ (a–c) and directed random networks $ER(N, p, q)$ (d–f). ID, OD, SD, PD, BC and TS represent the strategies based on in-degree, out-degree, sum of in-degree and out-degree, product of in-degree and out-degree and tabu search, respectively. Each quantity is an average over 100 independent realizations of network generation.

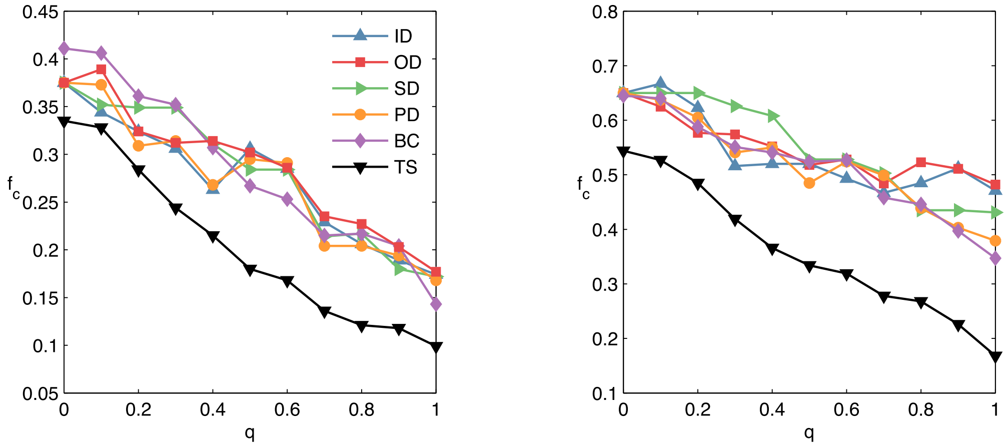
nodes leads to a deterioration of network performance and eventually to the disintegration of the network at a given critical removal fraction f_c. Smaller f_c implies more efficient network disintegration. Similar to the results in Fig. 4, we find that TS is obviously more destructive than other disintegration strategies. Moreover, we observe that the critical removal fractions of nodes f_c decrease with the increase of the proportion of one-way edges q for all disintegration strategies. It suggests that directed networks are more vulnerable than undirected networks.

To explore the difference between the disintegration strategies in directed networks and undirected networks, we consider the well-known terrorist network for Madrid train bombing (TNM), which involved many terrorist groups with members from different countries as a case study. The data of this network can be obtained from http://konect.uni-koblenz.de/ Here, TNM is purely considered to be a network composed with nodes (individual terrorists) and edges (possible connection between individuals). The initial network consists of 64 nodes and 243 two-way edges. We suppose that a certain proportion q of edges are directed and investigate the optimal disintegration strategies based on our proposed TS as shown in Fig. 6. We use the size of the largest strongly connected component as the measure function of network performance. The disintegration strength parameter n is 9. It is easy to see that the set of removed nodes are different in networks with various of q. In Fig. 6(a), the most of removed nodes have high betweenness centrality. However, the removed nodes tend to have lower betweenness centrality as the increase of q, which is shown in Fig. 6(b) and (c).

To be more precise, Fig. 7 shows the proportion of overlapping nodes (P_R) between the optimal disintegration strategy in directed networks with various values of q and the betweenness centrality strategy in undirected network with $q = 0$. We can observe that the optimal disintegration strategy in undirected network ($q = 0$) has high overlapping proportion with betweenness centrality, while the optimal strategies in directed networks ($q > 0$) have low ones. The similar results are observed with the degree strategies. It suggests that the critical structural properties in undirected networks, such as degree and betweenness centrality, have less ability to identify the important nodes in directed networks.

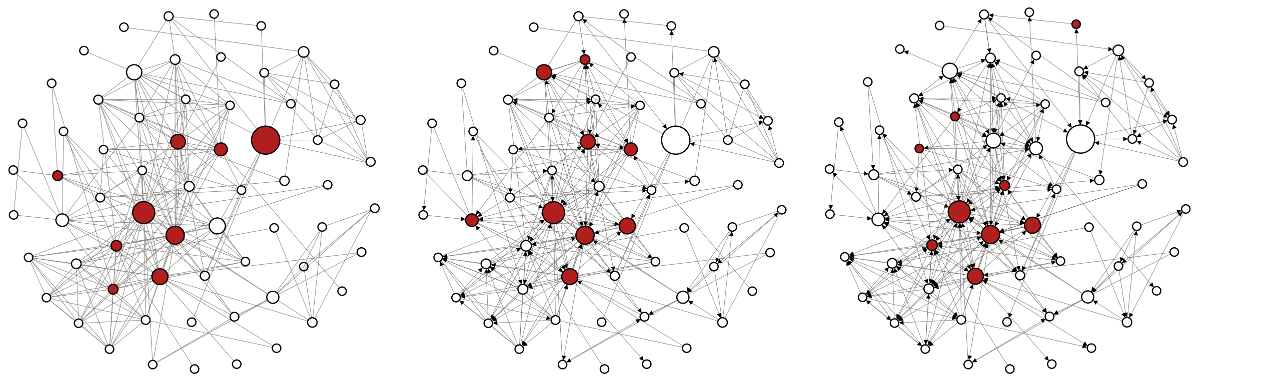
5. Conclusions and discussion

Disintegrating the directed networks is very important in many fields. Seeking optimal disintegration strategy among massive alternative strategies in directed networks is an important and challenging problem.



(a) CM(N = 1000, m = 3, λ = 2.5, q). (b) ER(N = 1000, p = 0.008, q).

Fig. 5. The critical removal fraction of nodes f_c versus the proportion of one-way edges q with various disintegration strategies in directed scale-free networks $CM(N, m, \lambda, q)$ (a) and directed random networks $ER(N, p, q)$ (b). ID, OD, SD, PD, BC and TS represent the strategies based on in-degree, out-degree, sum of in-degree and out-degree, product of in-degree and out-degree and tabu search, respectively. Each quantity is an average over 100 independent realizations of network generation.



(a) $q = 0$. (b) $q = 0.5$. (c) $q = 1$.

Fig. 6. The optimal disintegration strategies in Madrid Train Bombing terrorist network with $q = 0$ (a), $q = 0.5$ (b) and $q = 1$ (c). The size of node is proportional with its value of betweenness centrality.

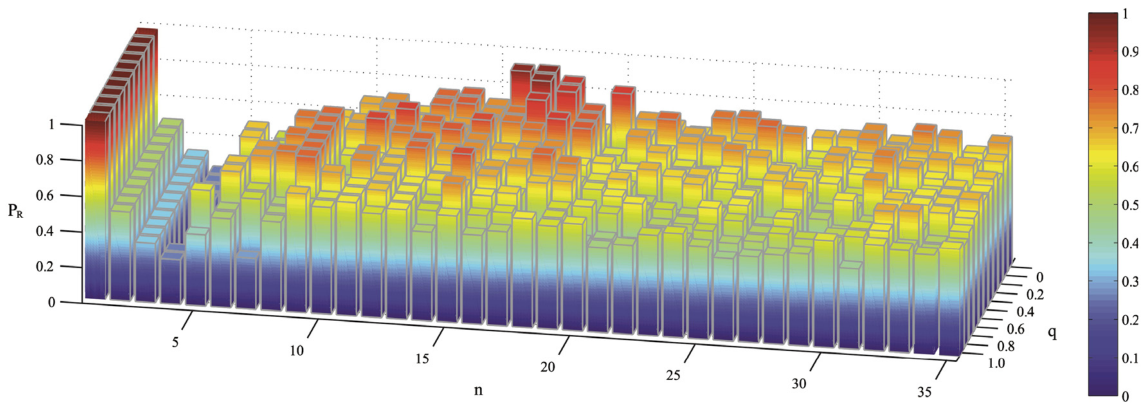


Fig. 7. The overlapping proportion of removed nodes between the optimal disintegration strategies in directed networks with various of q and the betweenness centrality strategy in undirected networks with $q = 0$.

In this paper, we have established an optimization model for disintegration strategy in directed networks, and solved this model by introducing the tabu search to get an optimal disintegration strategy. The destructive effect of this strategy has been verified by comparing with other typical centrality-based methods, such as degree-based and betweenness-based strategies. We have shown that the strategy based on tabu search can identify the approximate best group of nodes failure for maximum destructiveness of network and has an acceptable computational cost. In addition, the differences between the strategy based on tabu search and other strategies draw our attention. Those classical centrality metrics like degree and betweenness centrality which can identify the important nodes in undirected networks cannot recognize efficiently the really important nodes in directed networks, especially in networks with high proportion of one-way edges. Therefore, new structural properties in directed networks are expected to be defined as new attack criteria.

It is worth pointing out that, in comparison with centrality-based methods, the perfect information on the network structure is needed in our method. In many realistic cases, however, this perfect information is not available [36–38]. Thus the extension of our method to the case of incomplete or inaccurate information is expected in the future. Moreover, applying this disintegration method into network of networks is also an open and challenging problem.

Acknowledgments

This work was supported by the National Natural Science Foundation of China [grant number 71371185] and the Program for New Century Excellent Talents in University, China [grant number NCET-12-0141].

References

[1] M.E.J. Newman, The structure and function of complex networks, in: SIAM Rev, 2003, pp. 167–256.
[2] R. Albert, H. Jeong, A.L. Barabási, Error and attack tolerance of complex networks, Nature 406 (6794) (2000) 378–382.
[3] B. Bollobás, O. Riordan, Robustness and vulnerability of scale-free random graphs, Internet Math. 1 (1) (2003) 1–35.
[4] J. Wu, M. Barahona, Y.-J. Tan, H.-Z. Deng, Natural connectivity of complex networks, Chin. Phys. Lett. 27 (7) (2010) 078902.
[5] J. Wu, M. Barahona, Y.-J. Tan, H.-Z. Deng, Spectral measure of structural robustness in complex networks, IEEE Trans. Syst. Man Cybern. A 41 (6) (2011) 1244–1252.
[6] Z.-X. Wu, P. Holme, Onion structure and network robustness, Phys. Rev. E 84 (2) (2011) 026106.
[7] J. Gao, B. Barzel, A.-L. Barabási, Universal resilience patterns in complex networks, Nature 530 (7590) (2016) 307–312.
[8] B. Shargel, H. Sayama, I.R. Epstein, Y. Bar-Yam, Optimization of robustness and connectivity in complex networks, Phys. Rev. Lett. 90 (6) (2003) 068701.
[9] G. Paul, T. Tanizawa, S. Havlin, H.E. Stanley, Optimization of robustness of complex networks, Eur. Phys. J. B 38 (2) (2004) 187–191.
[10] A.X.C.N. Valente, A. Sarkar, H.A. Stone, Two-peak and three-peak optimal complex networks, Phys. Rev. Lett. 92 (11) (2004) 118702.
[11] J.G. Liu, Z.T. Wang, Y.Z. Dang, Optimization of robustness of scale-free network to random and targeted attacks, Modern. Phys. Lett. B 19 (16) (2005) 785–792.
[12] T. Tanizawa, G. Paul, R. Cohen, S. Havlin, H.E. Stanley, Optimization of network robustness to waves of targeted and random attacks, Phys. Rev. E 71 (4) (2005) 047101.
[13] A. Beygelzimer, G.E. Grinstein, R. Linsker, I. Rish, Improving network robustness by edge modification, Physica A 357 (3–4) (2005) 593–612.
[14] C. Schneider, A. Moreira, A. JS, S. Havlin, H. Herrmann, Mitigation of malicious attacks on networks, Proc. Natl. Acad. Sci. USA 108 (10) (2011) 3838.
[15] H.J. Herrmann, C.M. Schneider, A.A. Moreira, J. Andrade, S. Jos, S. Havlin, Onion-like network topology enhances robustness against malicious attacks, J. Stat. Mech. Theory Exp. 1 (1) (2011) 01027.
[16] R. Du, Robustness of network of networks under targeted attack, Phys. Rev. E 87 (5) (2013) 936–938.
[17] J. Gao, S.V. Buldyrev, H.E. Stanley, S. Havlin, Networks formed from interdependent networks, Nat. Phys. 8 (1) (2012) 40–48.
[18] J. Raab, H.B. Milward, Dark networks as problems, J. Public Administration Res. Theory 13 (4) (2003) 413–439.
[19] T. Kobayashi, K. Hasui, Efficient immunization strategies to prevent financial contagion, Sci. Rep. 4 (2014) 3834.
[20] R.M. Tripathy, A. Bagchi, S. Mehta, A study of rumor control strategies on social networks, in: International Conference on Information and Knowledge Management, 2010, pp. 1817–1820.
[21] A.P. Quayle, A.S. Siddiqui, S.J.M. Jones, Preferential network perturbation, Phys. A 371 (2) (2006) 823–840.
[22] J. Li, Y. Li, H.-Z. Deng, Y.-J. Tan, Optimal attack strategy in random scale-free networks based on incomplete information, Chin. Phys. Lett. 28 (6) (2011) 068902.
[23] S.Y. Tan, J. Wu, Lü L, M.J. Li, X. Lu, Efficient network disintegration under incomplete information: the comic effect of link prediction, Scientific Reports 6 (2016).
[24] P. Holme, B.J. Kim, C.N. Yoon, S.K. Han, Attack vulnerability of complex networks, Phys. Rev. E 65 (5) (2002) 056109.
[25] M. Bellingeri, D. Cassi, S. Vincenzi, Efficiency of attack strategies on complex model and real-world networks, Phys. A 414 (10) (2014) 174–180.
[26] L. Lü, D. Chen, X.-L. Ren, Q.-M. Zhang, Y.-C. Zhang, T. Zhou, Vital nodes identification in complex networks, Phys. Rep. 650 (2016) 1–63.
[27] M. Kitsak, L.K. Gallos, S. Havlin, F. Liljeros, L. Muchnik, H.E. Stanley, H.A. Makse, Identification of influential spreaders in complex networks, Nat. Phys. 6 (11) (2010) 888–893.
[28] E. Estrada, J.A. Rodriguez-Velazquez, Subgraph centrality in complex networks, Phys. Rev. E 71 (5) (2005) 056103.
[29] Y. Chen, G. Paul, S. Havlin, F. Liljeros, H.E. Stanley, Finding a better immunization strategy, Phys. Rev. Lett. 101 (5) (2008) 058701.
[30] C. Schneider, T. Mihaljev, S. Havlin, H. Herrmann, Suppressing epidemics with a limited amount of immunization units, Phys. Rev. E 84 (6) (2011) 061911.
[31] F. Morone, H. Makse, Influence maximization in complex networks through optimal percolation, Nature 527 (7579) (2015) 544.
[32] Y. Deng, J. Wu, Y. Tan, Optimal attack strategy of complex networks based on tabu search, Phys. A 442 (6) (2015) 74–81.
[33] S. Mugisha, H.J. Zhou, Identifying optimal targets of network attack by belief propagation, Phys. Rev. E 94 (1–1) (2016) 012305.
[34] J.R. Banavar, A. Maritan, A. Rinaldo, Size and form in efficient transportation networks, Nature 399 (6732) (1999) 130–132.
[35] F. Glover, Tabu search: A tutorial, Interfaces 20 (4) (1990) 74–94.
[36] J. Wu, Y.-J. Tan, H.-Z. Deng, D.-Z. Zhu, Vulnerability of complex networks under intentional attack with incomplete information, J. Phy. A 40 (11) (2007) 2665–2671.
[37] F. Taghavian, M. Salehi, M. Teimouri, A local immunization strategy for networks with overlapping community structure, Phys. A 467 (2016) 148–156.
[38] S.Y. Tan, J. Wu, L. Linyuan, M.J. Li, X. Lu, Efficient network disintegration under incomplete information: the comic effect of link prediction, Sci. Rep. 6 (2016) 22916.